A **tailored reading list** that integrates your PDF (Eberle), **Øksendal’s** classic *Stochastic Differential Equations*, and **Särkkä & Solin’s** *Applied Stochastic Differential Equations*, with a focus on:

* 📌 **Batch fermentation processes**
* 🧪 **System modeling with Julia’s SciML toolbox**
* ⚙️ **Simulation, inference, and uncertainty**
* 🧠 **Conceptual clarity with theory-backed insight**

**🧭 OVERVIEW: What each source is good for**

| **Source** | **Strengths** |
| --- | --- |
| **Eberle** | Deep **probability theory** and process construction. Excellent for understanding the foundations (Markov chains, martingales, Brownian motion). |
| **Øksendal** | Formal **SDE theory with Itô calculus**. Best for Itô's lemma, SDE formulation, existence/uniqueness theorems. |
| **Särkkä & Solin** | Practical + computational. Great for **simulations, estimation, Kalman filtering**, and **code examples (often in Julia/Python)**. Ideal for SciML-adjacent modeling. |

**🧑‍🔬 TAILORED READING PLAN**

**🧱 1. Foundations of Stochastic Processes**

Goal: Understand how randomness enters process dynamics.

| **Topic** | **Source** | **Sections** |
| --- | --- | --- |
| Conditional expectations & filtration | **Eberle** | 1.1–1.4 |
| Discrete-time Markov processes | **Eberle** | Ch. 2 |
| Martingales | **Eberle** | Ch. 3.1–3.2 |
| Brownian motion | **Eberle** | Ch. 4.1–4.3 |
| Gaussian processes (needed for Kalman filters) | **Eberle** | 4.2 |
| Gaussian processes & motivation | **Särkkä & Solin** | Ch. 3 |

**📉 2. SDEs: Theory and Practice**

Goal: Get fluent in Itô calculus, drift/diffusion forms, and simulation.

| **Topic** | **Source** | **Sections** |
| --- | --- | --- |
| Itô integrals, Itô calculus | **Øksendal** | Ch. 3 & 4 |
| Example: Geometric Brownian Motion | **Øksendal** | Ch. 4.2 |
| Simulation: Euler–Maruyama & Milstein | **Särkkä & Solin** | Ch. 4.1–4.3 |
| SDEs as time-continuous limits of Markov chains | **Eberle** | Ch. 5.3 |
| Implementing SDEs in Julia | **SciML Docs** | [StochasticDiffEq.jl](https://diffeq.sciml.ai/stable/types/sde_types/) (Euler–Maruyama, SDEProblem) |

**🔁 3. Application to Fermentation**

Goal: Model microbial/substrate/product systems with noise

| **Application** | **Source** | **Sections** |
| --- | --- | --- |
| Drift/diffusion design | **Øksendal** | Ch. 5.1–5.3 |
| Estimating parameters from data | **Särkkä & Solin** | Ch. 7 & 8 |
| Particle filtering, Kalman filter | **Särkkä & Solin** | Ch. 6–8 |
| Reaction kinetics + noise | **Your model** | via Julia's ModelingToolkit.jl or SDEProblem |
| Variability in growth/product rates | **You + Eberle** | Using concepts from Ch. 2 & 4 in custom drift/diffusion design |

**🧠 Pro Tips**

* Use **Eberle** to understand the *why* (foundations and measure-theoretic view).
* Use **Øksendal** to understand the *how* (SDE theory and calculus rigor).
* Use **Särkkä & Solin** to understand the *do* (simulate + fit to data + apply).